Seminole County Water Atlas Learning Kit
Dimensional Analysis *Handout*
Students gather data, select appropriate units of measure and convert from one system to another

Water Atlas Curriculum Lesson 8

UNITS are a critical part of describing every measurement. Before you can work with units mathematically, you frequently must convert from one unit to another. Dimensional analysis does not do your math for you, but it makes sure you get your multiplications and divisions straight. After that, all you have to do is find the conversion factors (unless you are the sort who memorizes numbers like 3.3, which I don't) and plug into a calculator (unless you are the sort who enjoys long division).

First visit www.Seminole.WaterAtlas.org > Water Resources Search Box on the right side of the page and select or type in the Lake Name. When the General Info page comes up, look over the information available on Seminole County's lakes. Notice that the units used are standard units i.e., acres, feet, gallons, etc. You will find these units and convert them to the appropriate metric units using dimensional analysis.

YOUR TASK Conversion to metric units is important because the scientific community uses only metric units in their measurements and journals and the use of standard measures is now limited to the U.S. and Bahrain internationally. Conversion units can be found at www.worldwidemetric.com. But first, look at the following information on using Dimensional Analysis for converting units.

WHATEVER YOU MEASURE, YOU HAVE TO USE UNITS. When we measure something, we always have to specify what units we are measuring in. For instance, if I tell you I am 16 tall, you haven't learned much about my height: your proper response would be "16 what?" If I respond 16 inches, you know that I am a very short person; on the other hand, if I tell you I am 16 feet, you can sign me up for the basketball team. And there are many other units of length I could use, such as meters and kilometers.

The above examples are for units of length, but there are many other things that we measure, and all of them require units. For instance, time can be measured in seconds or minutes (or hours, or days, etc); angles can be measured in degrees or in radians; and so on. Some units are made up of other units: for instance, you might measure the speed of a car in miles/hour. So "50 mi/hr" means that every hour, the car travels 50 miles; very different from a car traveling 50 mi/s, for instance! No matter what you are measuring, you must specify the units in order to give a measurement.

SOMETIMES YOU HAVE TO CONVERT BETWEEN DIFFERENT UNITS. Suppose that a 60-inch man stands on the head of a 6-foot man. How tall are they together? Obviously, you cannot find the answer to this question by adding 60 to 6. The reason is that the two figures are given in different units. Before you can add the two numbers, you have to convert one of them to the units of the other. Then, when you have two numbers in the same units, you can add them.

Seminole County Water Atlas Learning Kit
Dimensional Analysis *Handout*
Students gather data, select appropriate units of measure and convert from one system to another

Water Atlas Curriculum Lesson 8

In order to perform this conversion, you need a conversion factor. That is, in this case, you have to know how many inches make up a foot. You probably already know the answer: 12 inches is 1 foot. So knowing that, you can perform the calculation in two steps as follows:

a. 60 inches is really 60 / 12 = 5 feet

b. 5 feet + 6 feet = 11 feet

The second half of the calculation is easy; once all your units are the same, calculation is a snap. The tricky part is the conversion: because sometimes you have to divide, and sometimes you have to multiply. In the above example, I converted from inches to feet by dividing by 12. If I were going the other way, converting from feet to inches, I would have multiplied by 12. How do I figure out which to do?

One way is by using common sense. I know that 60 inches can't possibly be 720 feet; there have to be fewer feet than inches, obviously. However, sometimes common sense falls down on you. For instance, if you want to convert miles/hour into meters/second, what you do you multiply or divide by what?

Fortunately, there is another way: relatively easy once you get used to it, and guaranteed to work in every situation. This technique goes by the somewhat intimidating name of dimensional analysis, and I apologize. I didn't name it.

DIMENSIONAL ANALYSIS TELLS YOU WHAT TO MULTIPLY OR DIVIDE. Let's attack that problem I solved above, but use dimensional analysis, just so we can introduce the method. As you'll recall, the interesting part of the problem was step a), where I had to convert 60 inches into feet. I knew that there are 12 inches in a foot, so common sense told me to divide by 12. Here's the dimensional analysis way:

First of all, we write our conversion factor as a fraction. 12 inches = 1 foot, so the conversion factor is

$$\frac{12 \text{ inches}}{1 \text{ foot}} \ * \ \frac{1 \text{ foot}}{12 \text{ inches}}$$

Because 12 inches is the same thing as 1 foot, that fraction is just like 5/5; it's equal to 1. And we can multiply by it without changing anything.

So, let's try that on our 60 inches.

$$\frac{60 \text{ inches}}{1} \ * \ \frac{1 \text{ foot}}{12 \text{ inches}} \ = \ \frac{60}{12} \text{ ft.} \ = \ 5 \text{ ft.}$$

Name:

Date:

2

Seminole County Water Atlas Learning Kit
Dimensional Analysis *Handout*
Students gather data, select appropriate units of measure and convert from one system to another

Water Atlas Curriculum Lesson 8

The critical thing to note about the above equation is that *the units behave like numbers do when you multiply fractions.* That is, the inches on top and the inches on the bottom cancel out, leaving feet. Then all you have to worry about is the numbers.

Suppose that I had written in the wrong way, like the following.

$$\frac{60 \text{ inches}}{1} \;*\; \frac{12 \text{ inches}}{1 \text{ foot}} \;=\; 60 \text{ X } 12 \;\; \frac{\text{in. X in.}}{\text{ft.}} \quad \textbf{\textit{(wrong)}}$$

I would have immediately realized that multiplying 60 * 12 is wrong, because the units of the answer are inches * inches / feet instead of just feet. So the dimensional analysis shows me immediately that dividing is correct, and multiplying is not.

Such conversions are even easier in the metric system, of course, since all the conversion factors are multiples of ten. For instance, to convert 10 km to m, you would simply write

$$10 \text{ km} \;*\; \frac{1000 \text{ m}}{1 \text{ km}} \;=\; 10{,}000 \text{ m}$$

At this point, you may well be asking yourself "so what?" Dimensional analysis doesn't take much time, and it isn't very difficult, but it is certainly more trouble than the common-sense method.

To answer that, let's look at a more difficult unit conversion problem. Suppose that I want to convert 55 mi/hr into m/s. I know that 1 mile is 5280 feet, that 1 meter is 3.3 feet, that 1 hour is 60 minutes, and that 1 minute is 60 seconds. Now, what do I multiply, or divide, by what?

This is a very tricky question, but using dimensional analysis, it becomes a snap. First of all, I write all the above conversions as fractions, remembering that I might have to turn some of them upside-down.

$$\frac{1 \text{ mile}}{5280 \text{ ft}} \;*\; \frac{1 \text{ m}}{3.3 \text{ ft}} \;*\; \frac{1 \text{ hour}}{60 \text{ min.}} \;*\; \frac{1 \text{ min.}}{60 \text{ sec.}}$$

Now I write out an equation to convert from miles/hour to m/s, watching the units to make sure they cancel.

$$\frac{55 \text{ miles}}{1 \text{ hour}} \;*\; \frac{5280 \text{ ft}}{1 \text{ mile}} \;*\; \frac{3.3 \text{ ft}}{1 \text{ m}} \;*\; \frac{1 \text{ hour}}{60 \text{ min}} \;=\; \frac{55 * 5280 * 33 \text{ m}}{60 * 60 \text{ sec}} \quad \textbf{(wrong)}$$

Did you follow that, and make sure that the units cancel to give us meters/second? If you did, you hopefully were not surprised by the annotation on the right: because this equation is most

definitely incorrect. Do you see why? The "feet" on the top of the fractions don't cancel; so instead of being left with meters/seconds, we are left with the rather ungainly units of:

$$\frac{ft}{m} * \frac{ft}{s}$$

That is most definitely not what we are looking for; which is a perfect example of how, in a complicated problem like this, dimensional analysis can prevent you from making errors. If we turn the feet/meters fraction around, we get:

$$\frac{55 \; \cancel{miles}}{1 \; \cancel{hour}} * \frac{5280 \; \cancel{ft}}{1 \; \cancel{mile}} * \frac{1 \; m}{3.3 \; \cancel{ft}} * \frac{1 \; \cancel{hour}}{60 \; \cancel{min}} * \frac{1 \; \cancel{min}}{60 \; sec} = \frac{55 * 5280 \; m}{3.3 * 60 * 60 \; sec} = 24.4 \; m/sec$$

If you think that was overly difficult, try to imagine what it would have been like without dimensional analysis! You would still have had to multiple the 55, 5280, 3.3, 60, and 60, but first you would have had to figure out which to multiply and divide. And if you had made a mistake, how would you have caught yourself?

Use the above information to make the conversion calculations for several data sets on the Watershed Atlas site. Submit your information to be complied by the class to send to scientists who collect water data. Select Library Tab > Documents & Links to websites of agencies who test and measure the quality and quantity of our water.) You never know, your class may help bring the entire U.S. one step closer to catching up to the rest of the world.